

Professor Kyoung-SookMOON, PhD

E-mail: ksmoon@gachon.ac.kr

Gachon University

Heejean KIM, PhD

E-mail: heejean.kim@gmail.com

CK Goldilocks Asset Management

Professor Hongjoong KIM, PhD

E-mail: hongjoong@korea.ac.kr

Korea University

A PREDICTION METHODOLOGY FOR THE CHANGE OF THE VALUES OF FINANCIAL PRODUCTS

***Abstract.** A systematic algorithm based on data smoothing and the Bayes' theorem is proposed to predict the increase or decrease of a financial time series, which can be used in trading financial products when decisions need to be made between long and short positions. The algorithm compares the observed product values with those in the history to find a similar pattern with the maximum likelihood, based on which future up-down movement of the value is predicted. Empirical studies with S&P 500 Index and stocks of several companies show that the proposed methodology improves the rate of the correct predictions by about 30% or more, relative to naive prior probability or moving average convergence divergence predictions.*

***Keywords:** financial time series, numerical prediction method, empirical study, Bayes' theorem, maximum likelihood estimation, smoothing.*

JEL Classification: G17, C53, G11, C11

1. Introduction

One of the ultimate goals in finance is the accurate analysis of the financial market. In this paper, the analysis will focus on forecasting financial time series obtained from the values of financial products. In fact, such predictions are frequently needed when decisions are made between long and short positions in trading or portfolio optimizations. Since the financial market is complex and non-stationary, the problem is usually ill-posed and therefore such an analysis remains an effectively challenging task.

There have been numerous studies on mathematical or computational tools for those problems such as artificial neural networks (Kimoto *et al.*, 1990), support vector regression (Vapnik, 1998), or hybridization methods (Kuo *et al.*, 2001, Tsai and Hsiao, 2010). Even though these approaches work well in certain situations, Li (2010) shows that there is still a need for more efficient methods due to their requirements of heavy calculations or a large number of controlling parameters. Overfitting is another weakness of many traditional methods. Recently Karathanasopoulos *et al.* (2016) suggests the moving average convergence divergence (MACD) strategy as a simple reliable indicator for financial trends. Two moving averages are calculated with two different moving window lengths and the difference between longer and shorter moving averages is used for the prediction.

In this paper, based on data smoothing and the Bayes' theorem (Glimmet *al.* 2001), we propose a simple robust algorithm, which searches the reference history data for a pattern similar to the given observation using the maximum likelihood estimation. Based on the best matched history data, we make a decision on the up-down movement of the values. In order to filter out the noisy information of the data, we employ smoothing technique as well. Empirical data tests with S&P 500 Index as well as stock values of several companies show in Section 3 that the proposed algorithm improves the correct prediction rate for the up-down movement of the values by about 30% or more relative to the naive prior probability prediction or the moving average convergence divergence approaches.

The rest of this paper is organized as follows. We first set up the prediction problem and explain the main idea of the new prediction method based on the maximum likelihood estimation in Section 2. Then we describe the algorithm in detail including the smoothing process and moving window calculations in Section 3. Section 4 shows the empirical test results compared with naive prior probability and MACD-based predictions. Finally, conclusions and future works are given at the end.

2. Prediction Methodologies

Suppose that recent values of financial products have been observed and one is interested in whether the value increases or decreases in the future. For instance, at present time t_M we know the history of the values of an underlying

asset, $\{\dots, S(t_{M-1}), S(t_M)\}$ and would like to predict if the value $S(t_p)$ in near future, at t_p is larger than the current value, $S(t_M)$. Here the future time t_p can be a week or a month later than the present time t_M and the underlying asset can be a stock, an interest rate or a currency rate. Such a situation is frequently faced in trading or portfolio optimizations. In this study we focus on the prediction of stock values, although the same algorithm can be directly extended to other underlying assets as well.

One simple mathematical model is that these two events are equally likely so that the mathematical probabilities for the upward or downward movements are assumed to be both halves. Another well-known method is the moving average convergence divergence (MACD) formula. Two moving averages are calculated with two different moving window lengths. The difference between longer and shorter moving averages defines the MACD, which guides between long position and short position. Traditional 12 and 26 days are used in this study for shorter and longer window lengths of the moving averages of closing stock values. A 9-day moving average is provided as a trigger line. When MACD is above the 9-day average, a bullish position is taken. When MACD is below the 9-day average, a bearish position is taken. If this naive model (both upward and downward probabilities are 1/2) or the MACD approach can be improved through better analysis, it may lead to the increase of the profits from financial investments.

An enormous amount of stock market data is known these days and this history data includes all the properties of the underlying stock. So we may get an improved answer by extracting proper information, for instance looking for the similar pattern of recently observed up-down movement of the stock values in the history data. Based on this observation, the proposed algorithm performs following two steps. First, for given observed stock values a best match is found from the data in the history. Then the prediction is made based on the values of the best match found in the first step as will be explained below.

Let us consider a following simple mathematical algorithm based on the Bayes' theorem. Let \mathcal{M} denote a model for a stock value, which randomly varies in time. Glimm *et al.* (2001) describes Bayes' theorem

$$P(\mathcal{M}|\mathcal{O}) = \frac{P(\mathcal{O}|\mathcal{M})P(\mathcal{M})}{\int P(\mathcal{O}|\mathcal{M})P(\mathcal{M})d\mathcal{M}}$$

where \mathcal{O} represents an observation, used to improve the prior probability $P(\mathcal{M})$ of the model \mathcal{M} and $P(\mathcal{M}|\mathcal{O})$ is the posterior probability. The likelihood $P(\mathcal{O}|\mathcal{M})$ is the conditional probability of the observation \mathcal{O} given that the model is exactly \mathcal{M} . In this study, the maximum likelihood estimation approach is used for the prediction. That is, given observed stock values \mathcal{O} , the model \mathcal{M} needs to be found which maximizes the likelihood $P(\mathcal{O}|\mathcal{M})$ for the prediction. See Glimm *et al.* (2001) for more.

Let us consider discrete time moments, $t \in \{t_0, t_1, t_2, \dots\}$, and let \tilde{S}_i denote the stock value at $t = t_i$. Suppose that the stock values in the past N points in time $\{\tilde{S}_0, \tilde{S}_1, \dots, \tilde{S}_{N-1}\}$ are known for a sufficiently large N , which will be called the *history* values. Let us also suppose that stock values $\{\tilde{S}_{M-c+1}, \dots, \tilde{S}_M\}$ are observed at time t_M , which will be considered *given observation* for some M and c denotes the number of observed stock values. Based on the known history values, we want to predict whether the stock value increases or decreases in a near future, for instance, in p time units, i.e. we would like to know whether \tilde{S}_{M+p} satisfies $\tilde{S}_{M+p} > \tilde{S}_M$ or $\tilde{S}_{M+p} < \tilde{S}_M$. The basic idea of the algorithm is to find a *pattern* similar to the observed values from the history of known stock values. The c stock values $\tilde{S}_{[M-c+1, M]} \equiv \{\tilde{S}_{M-c+1}, \dots, \tilde{S}_M\}$ at time t_M are given as the observation $\tilde{\mathcal{O}}$ and the set of c stock values $\tilde{S}_{[k-c+1, k]} \equiv \{\tilde{S}_{k-c+1}, \dots, \tilde{S}_k\}$ for various k values constructs the reference history of models $\tilde{\mathcal{M}}$. Then $\tilde{\mathcal{O}}$ is compared to each one of $\tilde{\mathcal{M}}$ in the history as in Figure 1.

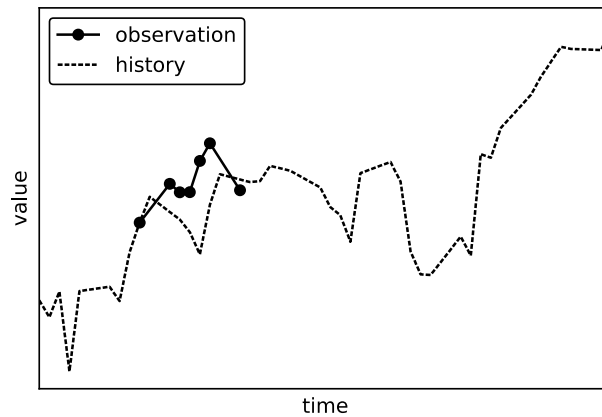


Figure1. The comparison of the observation $\tilde{\mathcal{O}} = \{\tilde{S}_{M-c+1}, \dots, \tilde{S}_M\}$ (circled) with each one of $\tilde{\mathcal{M}} = \{\tilde{S}_{k-c+1}, \dots, \tilde{S}_k\}$ for various k values in the history (dashed).

If the likelihood is maximized by, for instance $\tilde{S}_{[n-c+1,n]} \equiv \{\tilde{S}_{n-c+1}, \dots, \tilde{S}_n\}$ at time $= t_n$, i.e. if $\tilde{S}_{[n-c+1,n]}$ best matches \tilde{O} in the sense that the discrepancy between those two is minimized, then the prediction at t_{M+p} will be made the same as the value increases or decreases in p time units from t_n . That is, if the value increases at t_{n+p} compared to the value at t_n (i.e. $\tilde{S}_n < \tilde{S}_{n+p}$), the value at t_{M+p} will be also predicted to increase, while if the value decreases at t_{n+p} (i.e. $\tilde{S}_n > \tilde{S}_{n+p}$), the value at t_{M+p} will be predicted to decrease as well.

3. Improved Prediction Algorithm

In this section, the prediction strategy outlined above will be adapted to fit the financial data and led to an algorithm, which will be used to predict if a financial time series given by stock values will increase or decrease in a near future based on a history of *known* values in the past. Then we summarize the proposed algorithm.

Before measuring discrepancies between stock values from the observation and those from the history, a few preliminary steps need to be considered. Firstly, the direct comparison of the stock values for the discrepancy may trigger unexpected results because of the wiggly trends in the values. For instance, the deviation between the observation \tilde{O} and a model \tilde{M} may not be small enough due to oscillations even if they *do* have similar patterns. Such an oscillatory movement in stock values may cause false prediction. In order to resolve such a problem due to oscillations and filter out the noisy information from the data, smoothing of the data is proposed before one measures the discrepancies or predicts the future. In this study, a moving average filter (Johansson 2015) among many smoothing methods has been applied to stock values \tilde{S}_n , in which the output S_n is the median of the nearby m values, $\left\{\tilde{S}_{n-\frac{m-1}{2}}, \dots, \tilde{S}_{n+\frac{m-1}{2}}\right\}$ for some constant m . Then smoothed stock values from the observation are to be compared with smoothed stock values in the history.

Secondly, note that since the smoothed value S_n is the output from m values $\left\{\tilde{S}_{n-\frac{m-1}{2}}, \dots, \tilde{S}_{n+\frac{m-1}{2}}\right\}$, $\frac{m-1}{2}$ values in $\left\{S_{M-\frac{m-1}{2}+1}, \dots, S_M\right\}$ cannot be obtained at t_M because they depend on additional values $\left\{\tilde{S}_{M+1}, \dots, \tilde{S}_{M+\frac{m-1}{2}}\right\}$ which are not

available at t_M yet. Then, the observation at t_M has smoothed values up to $S_{M - \frac{m-1}{2}}$ only and one needs to predict the upward or downward movement of the stock value in $p + \frac{m-1}{2}$ time units in order to predict the movement at t_{M+p} . Thus, at t_M , c smoothed stock values $\mathcal{O} = \{S_{M - \frac{m-1}{2} - c + 1}, \dots, S_{M - \frac{m-1}{2}}\}$ will be compared with c smoothed values $\mathcal{M} = \{S_{k-c+1}, \dots, S_k\}$ in the history for various k values as in Figure 2.

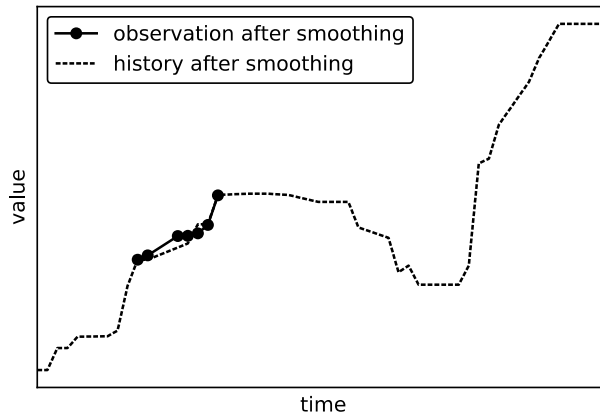


Figure2. The comparison of the observation after smoothing $\mathcal{O} = \{S_{M - \frac{m-1}{2} - c + 1}, \dots, S_{M - \frac{m-1}{2}}\}$ (circled) with each one of $\mathcal{M} = \{S_{k-c+1}, \dots, S_k\}$ for various k values in the history after smoothing (dashed).

Lastly, since only relative patterns are important, the observed stock values $\{S_{M - \frac{m-1}{2} - c + 1}, \dots, S_{M - \frac{m-1}{2}}\}$ in \mathcal{O} needs to be shifted when the discrepancy is measured so that the first value $S_{M - \frac{m-1}{2} - c + 1}$ matches that of $\{S_{k-c+1}, \dots, S_k\}$ in \mathcal{M} . That is, the discrepancy between the observation \mathcal{O} and a model \mathcal{M} is measured with respect to the L_1 norm with shift in this study, $\|\mathcal{O} - \mathcal{M}\| \equiv \sum_{i=-c+1}^0 \left| S_{M - \frac{m-1}{2} + i} - S_{k+i} - \tau \right| = \left| S_{M - \frac{m-1}{2} - c + 1} - S_{k-c+1} - \tau \right| +$

$\dots + \left| S_{M - \frac{m-1}{2}} - S_k - \tau \right|$ where the values in \mathcal{O} are shifted by $\tau = \tau(k) \equiv S_{M - \frac{m-1}{2} - c + 1} - S_{k - c + 1}$ so that the first values coincide as in Figure 2 as explained above. If \mathcal{O} best matches, for instance $S_{[n-c+1, n]} \equiv \{S_{n-c+1}, \dots, S_n\}$ at $t = t_n$, i.e. if the norm $\|\mathcal{O} - \mathcal{M}\|$ is minimized when $\mathcal{M} = S_{[n-c+1, n]}$, then the prediction will be dependent upon whether the stock value in the history increases or decreases in $p + \frac{m-1}{2}$ time units from t_n . That is, if $S_n < S_{n+p+\frac{m-1}{2}}$, then the stock value at t_{M+p} is also predicted to increase. If $S_n > S_{n+p+\frac{m-1}{2}}$, then the value at t_{M+p} is predicted to decrease as well. The algorithm for the up-down movement prediction is summarized in Algorithm 1 and will be called the MKK algorithm:

Algorithm 1 (MKK)

Require:

Set up the parameters: the widths for prediction (p), comparison (c), and smoothing (m)

Ensure:

For the index for observation $M = 1, 2, \dots$ do

Compute the smoothed observation data

$$\mathcal{O} = \left\{ S_{M - \frac{m-1}{2} - c + 1}, \dots, S_{M - \frac{m-1}{2}} \right\} \text{ at time } t_M$$

For the index for history $k = 1, 2, \dots, N$ do

Compute the smoothed history data

$$S_{[k-c+1, k]} = \{S_{k-c+1}, \dots, S_k\} \text{ at } t_k$$

Compute $\|\mathcal{O} - S_{[k-c+1, k]}\|$

EndFor

Find $S_{[n-c+1, n]}$ which minimizes $\|\mathcal{O} - S_{[k-c+1, k]}\|$

Decide the up-down movement at t_{M+p} by reading the value $S_{n+p+\frac{m-1}{2}}$

EndFor

4. Empirical Study

In this section, we test the accuracy of the proposed forecasting algorithm (MKK) by comparing its results with those from the naive or MACD-based predictions. The data used in the comparison is summarized in Table 1.

Table 1. Description of the data used in the comparison

Stock market index or stocks:	S&P 500 Index and stocks for Amazon, Apple, Bank of America, Citi Group, Coca Cola, Disney, Ford, GE, Microsoft, Nike
Data source:	Yahoo Finance (http://finance.yahoo.com)
Reference history data:	Values of the index or stocks for 5 years from 2009 to 2013
Observation data:	Values of the index or stocks for 2 years from 2014 to 2015

4.1 Effects of Parameters

In this section, we test wide ranges of values for the widths of prediction (p), comparison (c), and smoothing (m) to first study the effects of them on correct forecasts, then to identify the optimal ones. The values in the algorithm are summarized in Table 2.

Table 2. Description of parameters used in the algorithm

Width for Prediction (p):	$p \in \{1,2,3, \dots, 8\}$
Width for Comparison (c):	$c \in \{1,2,3, \dots, 8\}$
Width for Smoothing (m):	$m \in \{1,3,5, \dots, 13\}$

For each (p, c, m) and for each of the stock market index or stocks, the predictions are made for the observation data available between year 2014 and 2015. There have been total $8 \times 8 \times 7 = 448$ cases for (p, c, m) and 11 different

stocks including S&P 500 Index. And, there have been 488 predictions made when $p = 1$ and the number decreases by 1 as p increases. The computational simulation has been performed for all possible cases and only its summary is presented below for simplicity. The (average) rates of the correct predictions for each (p, c, m) are computed and then these rates are used as the measures for comparison in this section.

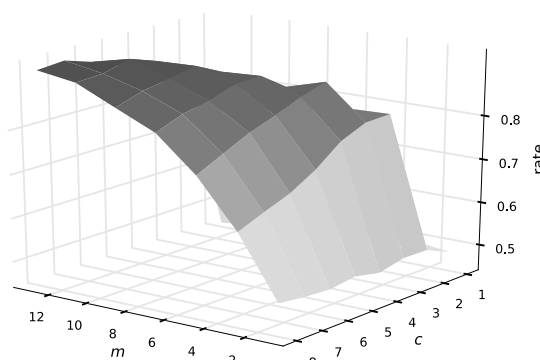


Figure3. The correct prediction rates when $p = 1$ as m and c change.

The correct prediction rate is maximized when the parameters are $p = 1$, $c = 6$, $m = 11$ and Figure 3 shows the rates when $p = 1$ as a function of m and c . Since $p = 1$ is too soon for certain actions after a bullish-vs.-bearish decision, the results in the neighborhood of $p = 2$, $c = 6$, $m = 11$ are shown in Figure 4 instead. Figure 4a at the top shows the correct prediction rates for various values of widths for prediction (p) with fixed parameter values $m = 11$, $c = 6$. The picture shows that as the duration p for the prediction becomes shorter, higher correct prediction rates are obtained, which is consistent with the intuition that one has better chance of predicting the values in nearer future.

Figure 4b in the middle shows the correct prediction rates for different widths of comparison (c) when the other parameter values $m = 11$, $p = 2$ are fixed. The picture shows that the more data we compare the higher correct prediction rates one obtains until one reaches the optimal point. Then the rate decreases slowly. That is, the comparison of too many values may not improve the accuracy. Figure 4c at the bottom shows the correct prediction rates when the width for smoothing (m) changes while the other parameter values are fixed to $c = 6$, $p = 2$.

As the width m increases, the correct prediction rates seem to increase up to the optimal point, then remain almost the same.

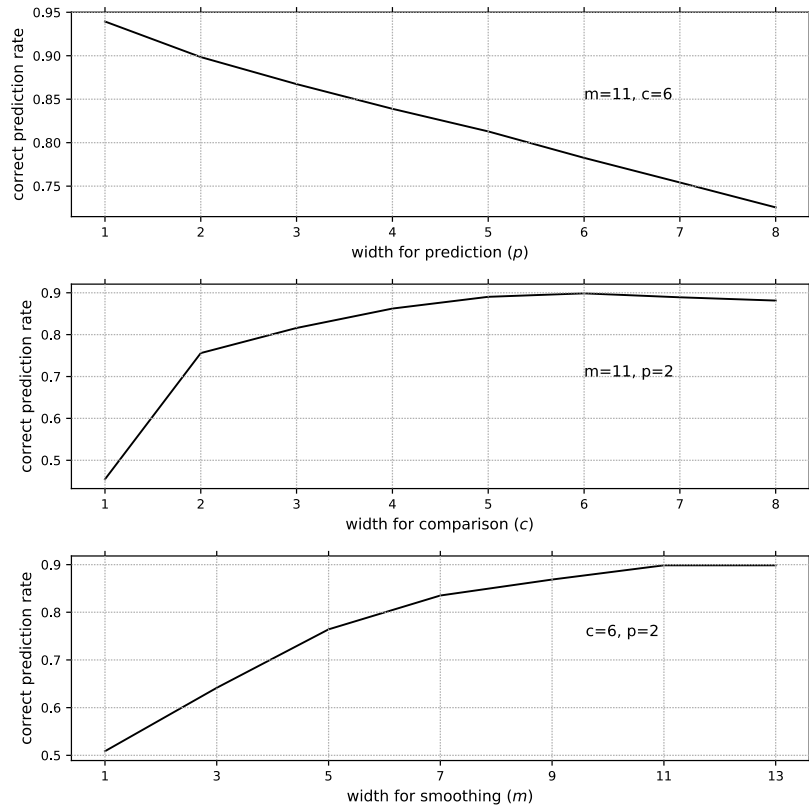


Figure 4. The comparisons of the correct prediction rates with respect to the parameters in the MKK algorithm near $p = 2$, $c = 6$, $m = 11$: Figure 4a (Top) $m = 11$, $c = 6$ with different widths (p) for prediction, Figure 4b (Middle) $m = 11$, $p = 2$ with different widths (c) for comparison, Figure 4c (Bottom) $c = 6$, $p = 2$ with different widths (m) for smoothing.

4.2 Prediction Results

In this section, let us consider the rates of the correct predictions for each (p, c, m, stock) for different stock values when the optimal values for the widths are used in the proposed MKK algorithm, that is, when the stock values are smoothed with 11 nearby values (i.e. $m = 11$) and $c = 6$ is used for the comparison of patterns. Figure 5 compares the prediction rates from the proposed MKK algorithm with $p = 2, 3, 4$ with those from the MACD formula and the prior probability. It shows that the rates from the MKK algorithm range mostly between 0.8 and 0.9 while that of the MACD approach is about 0.6 and that of the prior probability is only 0.5.

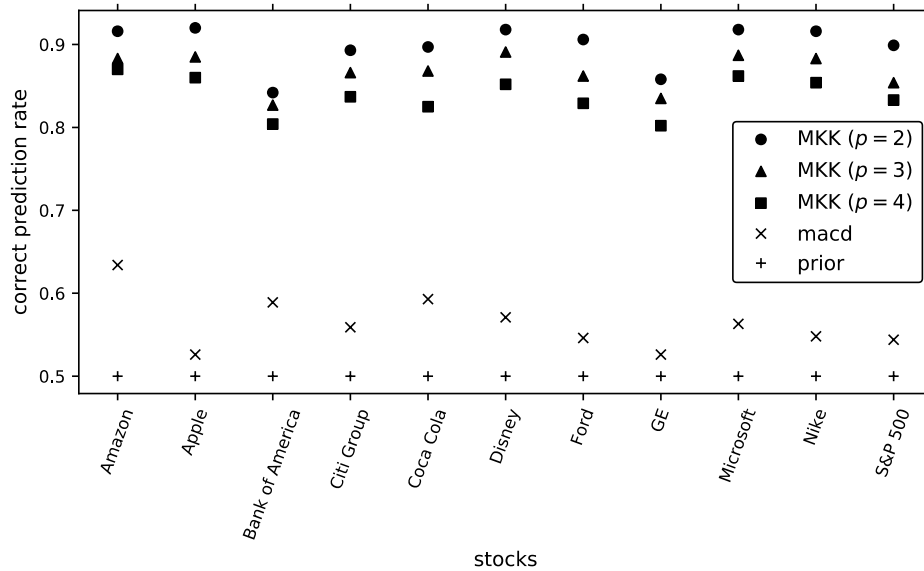


Figure 5. The correct prediction rates of the MKK algorithm with $p = 2$ (circle), 3 (triangle), 4 (square), the MACD formula (\times) and the naive prior probability (+) for various stocks when the observations from the data in 2014 and 2015 are tested with $m = 11$ and $c = 6$.

Table 3 shows the correct prediction rates from the MKK algorithm, the naive prior probability and the MACD formula for S&P 500 Index and several stocks when $m = 11$, $c = 6$, $p = 2$ are used. For instance, when stock values of Amazon are considered, 446 predictions from the proposed MKK algorithm out of total 487 predictions are correctly made while only 309 predictions from the MACD formula are correctly made so that the correct prediction rates from the

MKK and MACD algorithms are 91.6% and 63.4%, respectively, whose difference is 28.2%. When Apple is considered, the rate from the MKK algorithm increases up to 92.0%, which is 42.0% and 39.4% larger than predictions based on the prior probability and MACD formula, respectively. The overall average of the correct prediction rates of the MKK algorithm is about 89.8%, which is 39.8% more than that of the prior probability and 33.4% more than MACD formula.

Table 3. The comparisons of the correct prediction rates from the MKK algorithm with those of the naive prior probability and MACD formula out of 487 observations in 2014 and 2015 when $m = 11$, $c = 6$, $p = 2$ are used. In average 89.8% of predictions from the MKK algorithm are correctly made, which is 39.8% and 33.4% more than those of the prior probability and the MACD formula, respectively.

Company	MKK algorithm	prior probability (MKK – prior)	MACD algorithm (MKK – MACD)
Amazon	0.916	0.500 (0.416)	0.634 (0.282)
Apple	0.920	0.500 (0.420)	0.526 (0.394)
Bank of America	0.842	0.500 (0.342)	0.589 (0.253)
Citi Group	0.893	0.500 (0.393)	0.559 (0.334)
Coca Cola	0.897	0.500 (0.397)	0.593 (0.304)
Disney	0.918	0.500 (0.418)	0.571 (0.347)
Ford	0.906	0.500 (0.406)	0.546 (0.360)
GE	0.858	0.500 (0.358)	0.526 (0.332)
Microsoft	0.918	0.500 (0.418)	0.563 (0.355)
Nike	0.916	0.500 (0.418)	0.563 (0.355)
S&P 500	0.899	0.500 (0.399)	0.544 (0.355)
Average	0.898	0.500 (0.398)	0.564 (0.334)

Table 4 shows the results when $p = 3$, which is similar to that when $p = 2$. For instance, in case of Amazon, 429 predictions from the MKK algorithm out of 486 predictions are correct so that the correct prediction rate is 88.3%, which is only 3.3% lower than that when $p = 2$. The corresponding rate from the MACD formula is only 63.0%, which is 25.3% lower than that of the MKK method. The rates from the MKK algorithm range from 0.827 for Bank of America up to 0.891

for Disney and the average of the rates is 86.7%, which is 36.7% larger than that of the naive prior probability and 30.6% larger than the MACS formula.

Table 4. The comparisons of the correct prediction rates from the MKK algorithm with those of the naive prior probability and MACD formula out of 486 observations in 2014 and 2015 when $m = 11$, $c = 6$, $p = 3$ are used. In average 86.7% of predictions from the MKK algorithm are correctly made, which is 36.7% more than that of the prior probability and 30.6% more than MACD formula.

Company	MKK algorithm	prior probability (MKK – prior)	MACD algorithm (MKK – MACD)
Amazon	0.883	0.500 (0.383)	0.630 (0.253)
Apple	0.885	0.500 (0.385)	0.527 (0.358)
Bank of America	0.827	0.500 (0.327)	0.591 (0.236)
Citi Group	0.866	0.500 (0.366)	0.549 (0.317)
Coca Cola	0.868	0.500 (0.368)	0.595 (0.273)
Disney	0.891	0.500 (0.391)	0.572 (0.319)
Ford	0.862	0.500 (0.362)	0.539 (0.323)
GE	0.835	0.500 (0.335)	0.516 (0.319)
Microsoft	0.887	0.500 (0.387)	0.556 (0.331)
Nike	0.883	0.500 (0.383)	0.556 (0.327)
S&P 500	0.854	0.500 (0.354)	0.543 (0.311)
Average	0.867	0.500 (0.367)	0.561 (0.306)

5. Conclusion

A prediction algorithm for the up-down movement of a financial time series has been proposed, which works quite well when tested with real market data. It is expected that the proposed algorithm can be applied to a trading or hedging algorithm of the portfolio of risky assets and a real-time portfolio trading system algorithm as well, which are future research directions.

The proposed method can be seen as a generic technique to compute related problems in much more general context. The algorithm can be extended in the direction of currency or interest rates as well. In addition, the analysis of the effects of the moving average lengths is another future research direction.

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